

Indian Statistical Institute  
B.Math I Year  
Second Semester, 2005-2006  
Mid Semester Examination  
Probability Theory II

Time: 2 1/2 hrs

Date:01-03-06

Max. Marks : 80

Note: The paper carries 85 marks. Any score above 80 will be treated as 80.

1. Suppose the times it takes two students to solve a problem are independently and exponentially distributed with parameter  $\lambda > 0$ . Find the probability that the first student will take at least thrice as long as the second student to solve the problem. [12]
2. Let  $X, Y$  be i.i.d random variables with continuous strictly positive probability density function. Let  $F$  denote the common distribution function. Find the probability density function of  $F(X) + F(Y)$ . [15]
3. Let  $X_1, X_2$  be independent random variables such that  $X_1 \sim N(\mu_1, \sigma_1^2)$ ,  $X_2 \sim N(\mu_2, \sigma_2^2)$ . Let  $Y_1 = X_1$ ,  $Y_2 = X_1 + X_2$ . Find
  - (a) the distribution of  $(Y_1, Y_2)$
  - (b)  $\text{cov}(Y_1, Y_2)$  [13+5]
4. Let  $\mu_1, \mu_2 \in \mathbb{R}$ ,  $\sigma_1, \sigma_2 > 0$ ,  $-1 < \rho < 1$ .
  - (a) Write down the bivariate normal probability density function with means  $\mu_1, \mu_2$ , variances  $\sigma_1^2, \sigma_2^2$ , and correlation coefficient  $\rho$ .
  - (b) Find the marginal and the conditional probability density functions. [7+13]
5. Let  $X_1, X_2, X_3$  be independent  $N(0, 1)$  random variables. Indicating clearly the results you are using, find the distributions of
  - (i)  $X_1^2 + X_2^2 + X_3^2$ , (ii)  $\frac{2X_1^2}{(X_2^2 + X_3^2)}$ , (iii)  $\frac{X_1}{X_2}$  [6+6+8]